

Evaluation of the stability of an iron mining tailings dam

Avaliação da estabilidade de uma barragem de rejeitos de mineração de ferro

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Abstract: In engineering practice, the evaluation of dam stability is commonly done in a deterministic way, using Limit Equilibrium Methods. However, probabilistic methods have been increasingly accepted as a complementary analysis tool to represent the real condition of soil variability in the field. Thus, the article presents an evaluation of the stability of an upstream tailings dam, in the iron mining region of Minas Gerais, Brazil. In the research, the commercial program Slide 2D from the company Rocscience® was used. The stability studies adopted in the present work were based on the Limit Equilibrium Method proposed by Morgenstern-Price (1965). Probabilistic analyzes were carried out, using the classic methods of Monte Carlo (MC) and FOSM (First Order Second Moment), obtaining a deterministic factor of safety (FS) in the order of 1.06, for the condition of the previous dam rupture. The value of probability of rupture (Pr) by the FOSM Method was 27.0% (or 1:3.7), with a reliability index (β) equal to 0.621. Using the MC Method, Pr = 25.6% (or 1:3.9) and β = 0.705 were obtained. It appears, therefore, that both methods indicated a precarious condition of the stability of the dam, which broke in 2019.

Keywords: Geotechnics; Water security; Mineral extraction.

Resumo: Na prática de engenharia, a avaliação da estabilidade de barragens é comumente feita de maneira determinística, com uso de Métodos de Equilíbrio Limite. Porém, os métodos probabilísticos vêm tendo aceitação crescente como ferramenta complementar de análise, para representar a condição real da variabilidade dos solos no campo. Assim, o artigo apresenta uma avaliação da estabilidade de uma barragem de rejeito alteada para montante, na região de mineração de ferro em Minas Gerais, Brasil. Na pesquisa, utilizou-se o programa comercial Slide 2D da empresa Rocscience. Os estudos de estabilidade adotados no presente trabalho tiveram por base o Método de Equilíbrio Limite proposto por Morgenstern-Price (1965). Foram realizadas análises probabilísticas, utilizando-se os métodos clássicos de Monte Carlo (MC) e FOSM (First Order Second Moment), obtendo-se um fator de segurança (FS) determinístico na ordem de 1,06, para a condição da barragem anterior à ruptura. O valor da probabilidade de ruptura (Pr) pelo Método FOSM foi de 27,0% (ou 1:3,7), com um índice de confiabilidade (β) igual a 0,621. Pelo Método MC foi obtido Pr = 25,6% (ou 1:3,9) e β = 0,705. Verifica-se, portanto, que ambos os métodos indicaram uma condição precária da estabilidade da barragem, que veio a sofrer ruptura em 2019.

Palavras-chave: Geotecnica; Segurança hídrica; Extração mineral.

1. Introduction

Earth dams are essential structures in the evolutionary context of humanity, both for their role in economic development and in the promotion of new technologies to enable their construction (LEITE, 2019). Their use is quite diverse, with notable applications on mining tailings containment, irrigation, hydropower, water supply and flood control. According to the International Commission on Large Dams – ICOLD (2019), there are approximately 58,000 dams worldwide, 65% of which are earth dams.

Dam failures can cause extremely harmful environmental, economic and social impacts on society (HICKS, LI, 2018). Among the main factors that cause dam bursts, we may find foundation-related pathologies, inadequate spillway designs, slope instability, lack of erosion control, deficiencies in inspection and monitoring, and the absence of gradual safety mechanisms throughout the structure's lifecycle (ICOLD, 2001). When it comes to the causes of dam failures, the stability of upstream and downstream slopes is one of the most critical issues in the safety management of these structures. The various conditions that may lead to instability in these slopes must be analyzed.

Given the importance of dams, it is necessary to accurately identify and manage the risks to which these structures are subject (BOWLES *et al.*, 2011). In recent decades, there has been an increasing number of studies carried out to assess the safety condition of dams (HARIRI-ARDEBILI, 2018), particularly with regard to slope stability. Although most of these studies have focused on deterministic analysis, there is an upward trend toward the use of probabilistic methods to assess the stability of geotechnical structures, as evidenced by Sandroni and Guidicini (2022); Sousa (2021); Braga (2019); Oliveira (2018); Vecci (2018); Araújo (2018); Silva (2015); Sayão *et al.* (2012); Flores (2008); Fabrício (2006); Guedes (1997), Dell'Avanzi (1995); Sandroni and Sayão (1992).

Probabilistic approaches are important tools to quantify the uncertainties inherent in the complexity of earth dam construction (FERNANDES, 2020). They provide a range of benefits which guide the statistical calculus of terrain variability (soil and/or rock). They also corroborate more robust results, especially regarding the probability of failure (Pf) and the statistical distribution of dam factor of safety (FOS). Reliability indexes (β) are set so that complementary information is included in the single FOS obtained by traditional deterministic analysis (GUO X *et al.*, 2018).

Thus, this article presents a study on the use of probabilistic methods in analyses involving the safety of geotechnical structures and compares their results, based on a real case of an iron ore tailings dam raised using the upstream method.

2. Methodology

2.1 Dam type section

For the analyses conducted in this study, among the sections (Figure 1) established by the designer, the most unfavorable one in terms of stability was selected, that is, the one representing the greatest rock mass height, referred to as Section 4 in the project (TUV SUD, 2018), as shown in Figure 2.

The water table level (Figure 2) in the section at hand was defined via the analysis of devices installed in the structure, namely water level indicators and piezometers. The stability analyses are valid for this condition. If changes occur in the dam section hydrogeological information, reassessment is required.

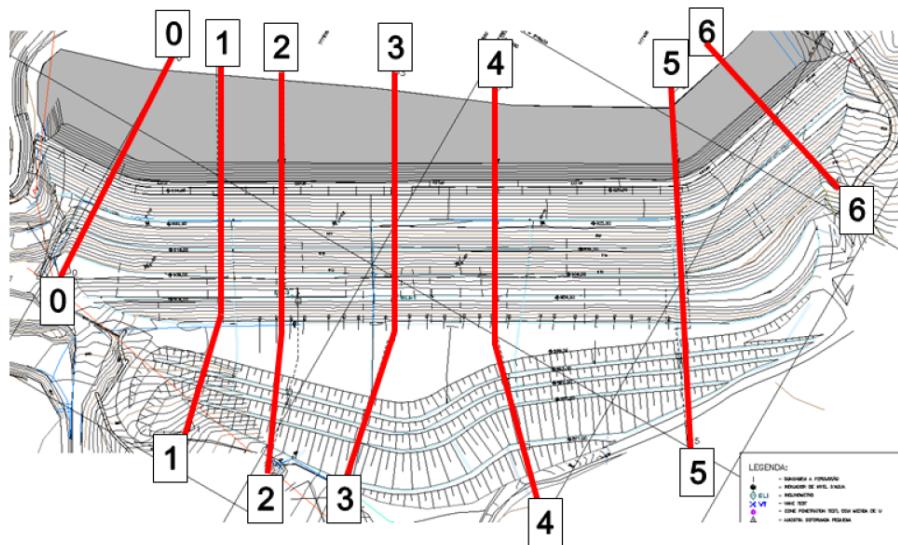


Figure 1 – Design sections for the dam under study.
Source: TUV SUD (2018).

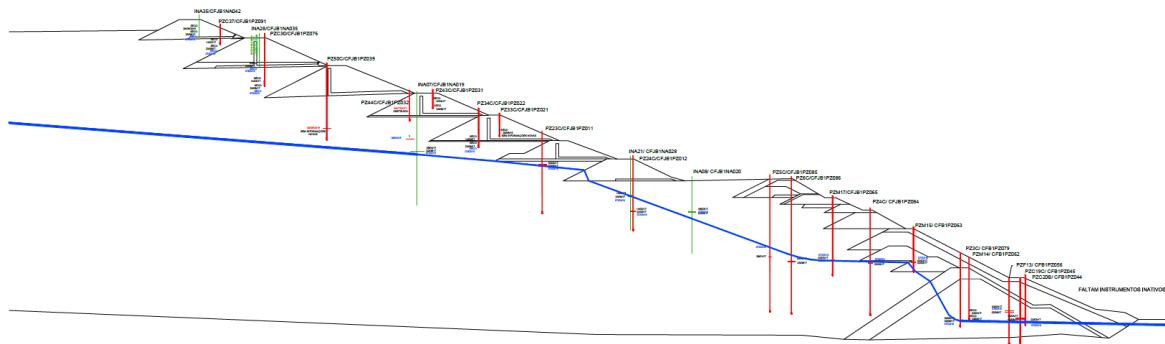


Figure 2 – Section 4 with the position of the water table.
Source: TUV SUD (2018).

2.2 Geotechnical parameters

The geotechnical parameters (Table 1) used in the deterministic and probabilistic stability analyses were defined via geological-geotechnical investigation campaigns and laboratory tests carried out over recent years. They are referenced in the “Revisão Periódica de Segurança de Barragem (RPSB)” (Dam Safety Periodic Review) document prepared in 2018 by TUV SUD.

Table 1 – Geotechnical Parameters Used in Stability Analyses

Material	γ (kN/m ³)	c' (kPa)	ϕ' (degrees)	$s_u/\sigma' v_0$	
				peak	residual
Residual Soil	20	16	30	-	-
Compacted Soil	19	10	30	-	-
Compacted Tailings	28	0	37	-	-
Ultrafine Ore	25	0	30	-	-
Saturated Tailings	26	-	-	0.26	0.09

Tailings	26	0	35	-	-
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Source: TUV SUD (2018).

In which:

γ - Specific weight in kN/m³;

c' - Effective cohesion in kPa;

ϕ' - Effective internal friction angle in degrees;

s_u/σ'_{v0} . Normalization of undrained shear strength.

2.3 Deterministic Stability Analysis

The stability analyses under undrained conditions were conducted using the Slide 2D software, version 9.0, by Rocscience®. They were processed using the Morgenstern-Price method (1965), based on limit equilibrium, adopting the Mohr-Coulomb failure model and assessing the failure surface from left to right. The effective parameters presented in Table 1 were adopted. The failure surface was automatically generated by the software.

2.4 Probabilistic Stability Analysis

The reliability of probabilistic stability is measured through a reliability index (β) and the probability of failure (P_f), which, according to Flores (2008), is defined as the probability of a (FOS) value being less than 1. Table 2 shows different levels of acceptance of P_f and β proposed by the US Army Corps of Engineers (1995) and Dell'Avanzi and Sayão (1998), respectively.

Table 2 – Reliability and Probability of Failure Indexes.

Performance level	Reliability Index (β)	Probability of failure (P_f)
High	5.0	3.0E-07
Good	4.0	3.0E-05
> Average	3.0	1.3E-03
< Average	2.5	6.0E-03
Poor	2.0	2.3E-02
Unsatisfactory	1.5	7.0E-02
Dangerous	1.0	1.6E-01

Source: USACE (1995); DELL'AVANZI, SAYÃO (1998).

According to Vecci (2018), there are several probabilistic methods for stability analysis, the most common are the First Order Reliability Method (FORM), the First-Order Second-Moment Method (FOSM), the Point Estimate Method (PEM) and the Monte Carlo Method (MC). In this work, the probabilistic methods used were FOSM and MC.

2.4.1 FOSM

According to Baecher and Christian (2003), the FOSM method is simple and may be divided into the following steps: identifying the variable parameters that affect the performance factor outcome; collecting statistical data, such as the mean, variance, and, if applicable, the correlation coefficient of the variables under study; calculating the performance indicator function result based on the average value of the properties of interest, by means of deterministic analyses; calculating the partial derivatives of the function for each variable; determining the influence of each variable on the function; calculating the function variance, the reliability index and the probability of failure of the performance indicator by applying the statistical data to the appropriate distribution for the problem; analyzing the results and the contribution of the parameters of interest to the factor of safety.

The factor of safety of a geotechnical structure is considered a performance function $FS(\bar{X})$ and is obtained from the mean (\bar{X}) of the geotechnical parameters ($x_1, x_2, x_3, \dots, x_n$). These parameters are defined as independent variables in the FOS calculation, such as cohesion, friction angle, specific weight and undrained shear strength, among others.

The expansion of the performance function $FS(\bar{X})$ through the Taylor series around the mean value (\bar{X}) is demonstrated in equation 1:

$$FS(X) = FS(\bar{X}) + \frac{FS'(\bar{X})}{1!} (X - \bar{X})^1 + \frac{FS''(\bar{X})}{2!} (X - \bar{X})^2 + \dots \quad equation\ 1$$

Where $FS'(\bar{X})$ and $FS''(\bar{X})$ represent, respectively, the first and second derivatives. After truncating equation 1 in the second moment of the series, that is, in the first-order derivative, and performing algebraic manipulation, the result is given by:

$$FS(X) - FS(\bar{X}) = FS'(\bar{X})(X - \bar{X})^1 \quad equation\ 2$$

By squaring equation 2 and, again, performing algebraic manipulation, the variance of the factor of safety is obtained, as shown in equation 3.

$$V[FS(X)] = (FS'(\bar{X}))^2 \cdot V(X) \quad equation\ 3$$

As it is known, determining the factor of safety involves several independent variables, not just a single one, as observed in equation 3. Therefore, according to Harr (1987), the actual variance (equation 4) of the FOS is calculated by adding the product of the square of the partial derivatives of the FOS related to each parameter taken as a variable and its respective variance.

$$V[FS] = \sum_{i=1}^n \left(\frac{\delta FS}{\delta x_i} \right)^2 \cdot V[x_i] \quad equation\ 4$$

The advantage of employing the FOSM method is the need to know only the moment values of the statistical distributions (mean and standard deviation) of the analyzed parameters (FALCÃO *et al.*, 2020). The main shortcoming of FOSM is that the requirement to solve partial derivatives may bring about complex and/or impossible developments.

Since solving these derivatives is not a simple procedure, authors such as Christian *et al.* (1992), Sandroni and Sayão (1992), Dell'Avanzi (1995), Vecci (2018) and Braga (2019) opted for the approximation of partial derivatives using the divided differences methodology, varying one parameter at a time, fixing the others, thus obtaining a FOS value for each parameter variation. As a result, for n independent variables, the FOSM method establishes $n+1$ analyses, that is, one for the mean values and n others to determine the derivatives $(\delta FS / \delta x_i)$ of each independent variable (FARIAS, ASSIS, 1998). The relationship determined by the division of the variation of the FOS value by the parameter (δx_i) variation which caused this FOS change supports the approximation of its partial derivative with respect to each parameter (VECCI, 2019), as indicated by equation 5.

$$\frac{\delta FS}{\delta x_i} = \frac{FS(x_i + \delta x_i) - FS(\bar{x}_i)}{\delta x_i} \quad equation\ 5$$

2.4.2 Monte Carlo Method

The Monte Carlo method is a statistical sampling technique that consists of generating a sufficient number of random values to adequately describe the probability distribution $F(X)$, so as to make the sample representative of the population. The accuracy of the estimate increases as the sample size and number of analyses performed approach infinity (VECCI, 2018). It is composed of the following steps (KRAHN, 2004): Determining an analytical method for stability analysis; specifying the input parameters as a probability distribution function; conducting N (equation 6) deterministic analyses of the system model and checking the eventual occurrence or violation of the limit state; calculating the reliability index and probability of failure based on the number of factors of safety less than one.

$$N = \left(\frac{h_{\alpha/2}^2}{4\varepsilon^2} \right)^n \quad equation\ 6$$

Where:

N - Minimum number of simulations for a given desirable (required) confidence level;

$h_{\alpha/2}$ - Index according to the desirable confidence level;

ε - Maximum error allowed (according to the confidence level);

n - Number of variables.

The index according to the desirable confidence level was determined by Harr (1987), as shown in Table 3.

Table 3 – Reliability indexes for normal distribution.

Confidence level (%) (1- α)	$h_{\alpha/2}$	Confidence level (%) (1- α)	$h_{\alpha/2}$
85.00	1.44	99.50	2.81
90.00	1.64	99.73	3.00
95.00	1.96	99.90	3.29
95.45	2.00	99.99	3.89
98.00	2.33	99.994	4.00
99.00	2.58	-	-

Source: HARR (1987).

Used to evaluate the safety of geotechnical structures, such as tailings dams, this method allows to re-edit the behavior of a dependent random variable (FOS), and to obtain the probabilities of failure, $P_f = P(FOS < 1)$, of a dam, provided that its geometry is available in scale, the mean, standard deviation and characteristic function of each variable parameter (γ , c , ϕ) that influences the factor of safety (VECCI, 2018).

For the application of the Monte Carlo method, a confidence level of 85% and three variables were considered, therefore, according to equation 2, the number of simulations (N) was 12231. The simulations were performed with Rocscience® Slide 2D software.

3. Results and discussion

3.1 Deterministic Analysis

As mentioned earlier, deterministic stability analyses were performed using the Morgenstern-Price method (1965). The factor of safety and the diagram with the failure surface are explained below.

It is observed in Figure 3 that, using the parameters presented in Table 3, the factor of safety under undrained condition (FOS = 1.058) is below the threshold indicated in Resolution 95/2022, issued by the regulatory and supervisory body for mining dams in Brazil, the National Mining Agency (ANM), whose minimum value is 1.1. In a study carried out with the same dam type section analyzed in this research, Braga (2019) obtained a FOS of 1,047, also via the Morgenstern-Price method.

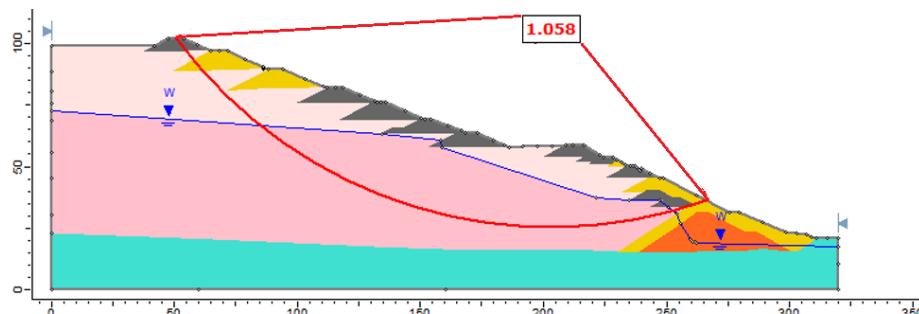


Figure 3 – Deterministic Stability Analysis under Undrained Condition.

Source: Authors (2023).

In light of the data reflected in both studies, it is noted that the FOS value found in our work is similar to the one found by the aforementioned author, which validates our deterministic stability analysis. The slight value variation is explained by the difference in software version.

The critical failure surface obtained exhibits global behavior, starting at the crest of the last raising and stretching along the tailings. However, it does not intercept the foundation, and exposes the critical safety conditions of the dam's downstream slope.

3.2 Probabilistic Stability Analysis

Through probabilistic stability analysis initially performed employing the FOSM method with $\pm 10\%$ variations for each mean parameter, as indicated by Dell'Avanzi (1995), it was possible to identify the geotechnical parameters with the greatest influence on the probability of failure. The highlighted variables were the $s_u/\sigma' v_0$ of saturated tailings, the tailings friction angle and the specific weight of saturated tailings, with influences of 94.81%, 2.58% and 1.93%, respectively. Thus, when performing the analysis by the Monte Carlo method, it was feasible to use the parameters that had greater interference in the Pf.

The result obtained regarding the probability of failure for the FOSM method was higher than that found for Monte Carlo, which corroborates the study carried out by Silva (2015), as well as by Vecci (2018) and Araújo (2018). The first two comprise probabilistic stability analyses in mine slopes and the third in dam.

The results of the analyses made by the FOSM and Monte Carlo methods were, in due order: [$\beta=0.621$; $Pf=27\%$] and [$\beta=0.705$; $Pf=26\%$]. The probabilities of failure and the reliability indexes obtained for both methods (FOSM and MC) fall within the "dangerous" range, both for the values proposed by USACE (1995) and by Dell'Avanzi and Sayão (1998), presented in Table 2.

4. Final Remarks

For this study, the β value obtained by MC (0.705) was slightly higher than the one found by FOSM (0.621). However, such a difference only resulted in a 1% impact on the probability of failure. Therefore, both probabilistic analysis methods indicated precarious stability conditions for the dam, which came to burst in 2019.

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