Relações e Nominalismo: Os Limites das Abordagens Baseadas em Teoria de Classes

Relations and Nominalism: The Limits of Class-Theoretic Approaches[1]

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Resumo: Neste artigo, discutirei um problema para as teorias nominalistas: qual é o status ontológico das relações? Como os nominalistas rejeitam os universais, eles devem reduzir relações a particulares. Algumas teorias, como o Nominalismo de Classes e o Nominalismo por Semelhança, postulam classes para reduzir as relações. Examino duas críticas clássicas à análise das relações no Nominalismo: o problema da ordem e as questões envolvendo o procedimento de Wiener-Kuratowski. Embora a tese de que as relações não são classes seja amplamente aceita, apresento dois argumentos adicionais contra essa posição. Em primeiro lugar, a determinação de qual classe corresponde a uma relação é arbitrária. Em segundo lugar, argumento que as teorias nominalistas não

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conseguem fornecer uma estrutura hierárquica adequada da realidade. Enquanto objeções anteriores se concentram principalmente em questões técnicas das reduções de relações para classes, minha crítica questiona a coerência metafísica dessas reduções.

Palavras-chave: Relações; Nominalismo; Classes.

Abstract: In this paper, I shall discuss a problem for nominalist theories: What is the ontological status of relations? Since nominalists reject universals, they must reduce relations to particulars. Some theories, such as Class Nominalism and Resemblance Nominalism, postulate classes to reduce relations. I examine two classical criticisms against the analysis of relations in Nominalism: the problem of order and the issues involving the Wiener-Kuratowski procedure. Even though the claim that relations are not classes is a well-established thesis, I introduce two additional arguments against it. First, determining which class corresponds to a relation is arbitrary. Second, I argue that nominalist theories fail to provide an adequate hierarchical structure of reality. While previous objections focus primarily on technical issues in the reduction of relations to classes, my critique challenges the metaphysical coherence of such reductions.

Keywords: Relations; Nominalism; Classes.

Properties may be distinguished by means of arity: there are monadic properties and relations with arity greater than one. Properties can also be categorized by means of orders: first-order properties, second-order properties, and so on. First-order properties characterize objects—for example, *being wise* characterizes *Socrates*. Second-order properties characterize first-order properties—for example, *being a virtue* characterizes *being wise*. The same holds for relations: there are first-order relations, second-order relations, etc.

A theory is nominalist if and only if it explains properties without positing universals. The goal of this paper is to investigate a problem involving the ontological status of relations in Nominalism. More precisely, I will discuss the reduction of relations to classes, as proposed by Class Nominalism and Resemblance Nominalism. I will defend and improve a well-known criticism of this kind of reduction.

That relations are not classes is widely defended in the literature. Historically, relations were identified not merely with classes in general, but with ordered classes due to non-symmetric relations. Some objections have been raised against this reduction. One well-known objection arises from the fact that ordered classes can be reduced to non-ordered classes. For instance, (a, b) may be reduced to {{a}, {a, b}}. However, this reduction is not univocal. The same ordered class corresponds to different non-ordered classes: (a, b) may be reduced to both {{a}, {a, b}} and {{b}, {a, b}}. Therefore, the identification of a relation with a single non-ordered class is always arbitrary.

In this paper, I shall provide additional arguments to establish that relations are not classes. I will demonstrate that non-symmetric relations are not necessary to generate this problem. Identifying a relation with a class is an arbitrary procedure because collecting objects that compose this class lacks a definite criterion.

In section (1), I discuss Class Nominalism. The central task here is to provide an argument against the explanation of relations. The problem of order and the Wiener-Kuratowski reduction are investigated. Additionally, I discuss the problem of whether nominalists could propose an adequate hierarchical structure of reality.

At first glance, Resemblance Nominalism seems to be a better theory than Class Nominalism, as resemblance nominalists do not identify relations with classes. However, this theory postulates classes to account for relational facts. I claim that introducing classes at this point generates the same problems. In section (2), the main goal is to discuss Rodriguez-Pereyra's arguments in favor of classes as an explanation for relational facts. I argue that choosing classes to account for relational facts is inevitably arbitrary.

1. Class Nominalism

Set theory is an important branch of mathematics and experienced significant development in the twentieth century. It is in a better position than any theory of properties for several reasons. First, the identity conditions of classes are entirely determined by the principle of extensionality. Properties, on the other hand, do not exhibit precise identity conditions. Second, the connection between particulars and universals is far less clear than the connection between classes and their members. Plato, for example, appeals to a metaphor—participation—to explain this relation between particulars and universals. In Class Nominalism, this relation is simply defined in terms of membership. In this way, Class Nominalism can easily solve classical problems, such as the problem of predication and the problem of abstract reference (Loux and Crisp, 2017, p. 21–30). Therefore, Class Nominalism is an attractive solution to the Problem of Universals. However, Armstrong has formulated criticisms against this theory (Armstrong, 1989, p. 25–36). In this section, I investigate one of these criticisms: Class Nominalism cannot provide a correct explanation for relations.

In Class Nominalism, properties are identified with classes of particulars. Since relations are also properties, albeit binary, they are reduced to classes of particulars as well. Appealing to classes to explain unary properties does not pose a problem *per se*. However, properties with arity larger than one, i.e. relations, generate specific problems for Class Nominalism.

Relations can be classified as symmetric and non-symmetric:

(i) *R* is a symmetric relation if and only if $\forall xy(Rxy \rightarrow Ryx)$ is true;

(ii) otherwise, *R* is a non-symmetric relation[2].

Consider two relations: *precede* (P) and *succeed* (S). They are non-symmetric relations, but they may be represented by the same class:

(iii) $\{\{x, y\}/Pxy\} = \{\{a, b\},\{b, c\}, ...\}$

(iv) $\{\{x, y\}/Sxy\} = \{\{b, a\},\{c, b\}, ...\}$

These classes are composed of non-ordered classes. Since {a, b} and {b, a} are the same class, and so are {b, c} and {c, b}, (iii) and (iv) cannot be differentiated. Thus, (P) and (S) would be the same relation from this perspective.

(P) and (S) are non-symmetric relations. In this kind of relations, members must be arranged by order. Class Nominalism can appeal to Set Theory to solve this problem: relations are defined in terms of ordered pairs. Ordered pairs must obey the following rule:

(v) $(x, y) = (u, v)$ if and only if $x=u$ and $y=v$.

Now, (P) and (S) can be reduced to different classes:

(vi) $\{(x, y)/Pxy\} = \{(a, b), (b, c),...\}$

[2] Asymmetric relations can be differentiated in a similar manner. *R* is an asymmetric relation if and only if $\forall xy(Rxy \rightarrow \neg Ryx)$ is true. The difference between non-symmetric and asymmetric relations is not important for this work.

(vii) {(x, y)/Sxy} = {(b, a), (c, b),...}

There are two main criticisms of this solution. First, two classes are used to differentiate (P) and (S). However, it is not clear why (P) must be reduced to (vi) and (S) to (vii) and not vice-versa. The order of the elements of subclasses in (vi) is conventional; it is just another class of ordered pairs, like (vii), but with a different order rule. Choosing (vi) or (vii) as the correct class for (P) or (S) is entirely a matter of convention.

Second, an ordered pair is essentially defined by its order. Russell (1975, p. 67) remarked that ordered classes have a hidden intensional aspect if we want to distinguish the ordered pair (x, y) from the ordered pair (y, x). Therefore, (vi) and (vii) do not really explain the relations (P) and (S).

Class nominalists can still defend that relations are non-ordered classes. Ordered pairs are defined through non-ordered classes. Norbert Wiener has formulated the first definition of ordered pair in 1914 (Enderton, 1977, p. 36):

(viii) (x, y)={{{x}, Ø}, {{y}}}.

However, a simpler and better definition was provided by Kuratowski. This definition is standard in the present day:

(ix) (x, y)={{x}, {x, y}}.

Of course, both definitions satisfy the identity criterion of an ordered pair.

Class Nominalism is committed to concrete particulars and classes. If relations are reduced to non-ordered classes, the same commitment is maintained.

Even though definitions (viii) and (ix) satisfy the identity criterion of an ordered pair, there are criticisms of the assumption of Class Nominalism that relations are classes. In the literature, two objections have been made against reducing relations to classes: (a) (viii) and (ix) are arbitrary reductions and (b) the problem of order is not eliminated. The first problem concerns the conventional aspect of (viii) and (ix). Quine (1960, §53) points out that many definitions of ordered pairs are equally valid. Consider the relation *precede* (P). This relation should be reduced to a class of ordered pairs (vi). Since there are many definitions of ordered pairs, we can translate (P) into several non-ordered classes. Take, for example, two reductions of (P) through (viii) and (ix):

(x) {(x, y)/Pxy} = {{{{a}, ∅}, {{b}}}, {{{b}, ∅}, {{a}}}, {{{c}, ∅}, {{d}}}, {{{d}, ∅}, {{c}}},...}

(xi) {(x, y)/Pxy} = {{{a}, {a, b}}, {{b}, {a, b}}, {{c}, {c, d}}, {{d}, {c, d}},...}

Shall the relation (P) be accounted for as (x) or (xi)? This problem is similar to an argument formulated by Benacerraf (1965). He argues that numbers cannot be sets: since different sets, such as {{∅}} and {∅, {∅}}, are appropriate to account for number two, there is no independent criterion for choosing which set corresponds to each number. Thus, identifying numbers with sets is misleading.

In the literature, some philosophers apply Benacerraf's argument to relations, such as Kitcher (1978) and Williamson (1985, p. 254–255). In Set Theory, a relation can be defined as an arbitrary class. This does not generate a specific problem. Choosing one of them as the standard definition[3] may solve

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[3] In fact, this problem does not arise in Set Theory. Even though we use the same word "relation" in Set Theory and Ontology, two different meanings should be distinguished. Consider two classes: A and B. A × B is the cartesian product between classes A and B. A relation between these classes is a subclass of A × B. In Ontology, we assume a pre-theoretical meaning of relation; it is an intensional notion. Solutions to the problem of universals try to eliminate this notion by assuming at least one ontological category

the problem. In Ontology, however, it is not so simple. Ontological inquiry tries to determine what exists and which kinds of entities are fundamental. (P) is reduced in two distinct ways in (x) and (xi). They are different ontological solutions. In fact, other explanations of relation (P) are possible because ordered pairs can be reduced to non-ordered classes in multiple ways. There is no independent criterion for choosing which class should represent this relation. Therefore, Class Nominalism's reduction of relations to classes is arbitrary.

The second issue concerns the order problem [4]. Hochberg[5] (1981) claims that order is not eliminated by the Wiener-Kuratowski procedure. If we combine Class Nominalism with this procedure, the reductions of relations (P) and (S) are, respectively:

(xii) {{{a}, {a, b}}, {{b}, {b, c}}, {{c}, {c, d}},...}

(xiii) {{{b}, {a, b}}, {{c}, {b, c}}, {{d}, {c, d}},...}

However, Hochberg (1981, p. 162) points out that identifying relation (P) with class (xii) or (xiii) is arbitrary. One may assume a rule that reduces (P) to (xii) or a rule that reduces (P) to (xiii). In any case, an implicit order of the elements of the class (xii) is assumed.

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as fundamental. Class Nominalism identifies each property with a different class. I have argued that identifying relations with classes is not an easy task.

[4] The problem of order in relational facts is not exclusive to Nominalism (Tegtmeier, 2004).

[5] He assumes that this analysis can be understood as an interpretation of Russell. Imaguire (2006, p. 164–165) argues that Russell anticipates the problems of the Wiener-Kuratowski procedure.

A central difficulty for Class Nominalism is the need to introduce ordered pairs to explain non-symmetric relations. In the literature, another topic on relations can impact this debate. According to some philosophers, order is not characteristic of certain relations. (Williamson, 1985; Fine, 2000; Dorr, 2004). Even if there were no non-symmetric relations, I claim that the explanation offered by Class Nominalism fails for two reasons. First, the explanation proposed by Class Nominalism provides a strange ontological hierarchical structure. Second, Benacerraf's argument can be applied even if ordered pairs are not used to account for relations.

As I stressed at the beginning of this paper, reality seems to have a hierarchical structure: objects, first-order properties and relations, second-order properties and relations, and so on. Nominalist theories try to explain properties without postulating universals. Ostrich Nominalism, for example, assumes only concrete particulars to solve the problem of universals—reality is not hierarchical.

Class nominalists could assume two strategies. First, it is possible to defend a reductive approach. There are only first-order properties and relations. Higher-order properties should be reduced to first-order properties. However, reductionist approaches are often controversial. The possibility of reducing second-order properties to first-order properties depends on another question. Some philosophers try to paraphrase sentences to avoid higher-order properties. But classes could also be used to provide a structured world without eliminating second-order properties. I argue now that in both cases relations continue to be a problem.

First, let us see the hierarchical approach. Class Nominalism must provide a solution for higher-order properties. Consider the second-order property of *being a color*. It characterizes first-order properties, for example: *being blue*, *being green*, etc. According to Class Nominalism, second-order properties could be reduced to classes of classes. Thus, the second-order property of *being a color* could be explained as follows:

(a) {class of blue things, class of green things,…}

As realist theories, Class Nominalism can exhibit a hierarchical ontological structure: objects, classes of objects, classes of classes of objects, and so on.

Two hierarchies could be generated: one with properties and one with classes. Intuitively, in Class Nominalism, these hierarchies should stand in one-to-one correspondence. Unary properties follow this intuition: first-order properties correspond to classes of objects, second-order properties correspond to classes of classes of objects, and so on. However, the same does not hold for relations. First-order relations correspond to classes of classes of objects, placing them at the same level in the class hierarchy as second-order properties. This result is counterintuitive because both first-order properties and first-order relations characterize objects. It would be more natural to expect them to occupy the same level in the class hierarchy, while first-order relations and second-order properties should be at different levels.

Even though objects, classes of objects, and classes of classes are all particular entities, their positions in the class hierarchy are distinct. This hierarchy does not imply that objects, classes, and classes of classes are ontologically different. Since first-order properties and first-order relations characterize objects, reducing them to classes should maintain that they occupy the same level in the class hierarchy. However, if properties and relations of the same order appear at different levels (and first-order relations share the same level with second-order properties), then the way Class Nominalism explains properties and relations is problematic.

This problem can be avoided if Class Nominalism assumes a reductionist approach to second-order properties and relations. In this approach, reality is composed of three levels: objects, classes (which represent unary properties) and classes of classes of objects (which represent relations). This would work if all relations were symmetric. Unfortunately, there is an additional problem.

To reduce unary first-order properties to classes is an easy task. There is a clear rule for constructing classes that represent these properties. Consider the property of *being human* (H). Every object that has this property belongs to the natural class of human beings:

(i) H = {a, b, c, d, e,…}

Relations are not so simple because the rule for constructing classes cannot be the same. Consider a possible world *w* with only three objects: *a*, *b*, and *c*. These objects have one property *Px* and they are related by a symmetric relation *Rxy*. World *w* is composed of three unary facts—*Pa*, *Pb* and *Pc*, and three relational facts—*Rab*, *Rac* and *Rbc*. If the objects that instantiate *Px* are collected, property *P* could be identified with the following class:

(ii) {a, b, c}.

If the same rule is applied to properties and relations, property *P* and relation *R* are reduced to the same class (ii). The objects that saturate Rxy in this possible world are just picked up by the same class.

Of course, relations are not reduced in this way. Since *w* is composed of three relational facts, class nominalists could argue that the objects of each fact should be collected separately:

(iii) {{a, b}, {a, c}, {b, c}}.

Ordered pairs are not necessary because we are assuming that there are no non-symmetric relations. In *w*, there are three unary facts. The same procedure to pick up objects of each unary fact separately can be used:

(iv) {{a}, {b}, {c}}.

There is no non-arbitrary rule about 'the' correct way to construct classes that correspond to *P* and *R*. Which is the correct analysis of property *P*: (ii) or (iv)? Which is the correct analysis of relation *R*: (ii) or (iii)? Since *P* and *R* could be differentiated by means of the classes (ii) and (iii), class (iv) is not postulated to reduce *P*. This should not be a problem for Set Theory. However, in terms of ontological theory, some criticisms could be made against this proposal. It seems that reducing *R* to (iii) is completely arbitrary.

The introduction of class (iii) could be considered an arbitrary maneuver. Initially, there is a rule to construct unary properties: collect all particulars instantiated by a property and put them together in a class. Unfortunately, this rule cannot be applied to relations; they have to obey a different rule. Particulars of each relational fact should be collected in a class. In a binary relation, there are *n* classes of two elements for *n* relational facts. Thus, a binary relation would be reduced to a class of classes of two elements. This rule needs to be introduced in order to eliminate the problem. *P* and *R* should be distinguished in an ontology of classes. Without a rule to construct class (iii), *P* and *R* could not be distinguished.

Consider another rule to construct different classes for *P* and *R*. Assume that there are only first-order properties. One may propose a hierarchical rule to define properties and relations: unary properties correspond to classes of objects; binary relations correspond to classes of classes of particulars; and so on. Assume that there is a ternary relation *T* in *w* and a relational fact *Tabc*. *P*, *R*, and *T* could correspond to the following classes, respectively:

(v) {a, b, c}

(vi) {{a, b, c}}

(vii) {{{a, b, c}}}

Which class should the relation *R* correspond to: (ii), (iii) or (vi)? Choosing a criterion to decide this question is not an easy task. Even though there were only symmetric relations and, therefore, all problems involving ordered pairs would be removed, there is an indeterminacy problem in reducing relations to classes. Benacerraf's argument could be applied once again in this case: relations could not be classes because there are manifold ways to reduce relations to classes.

2. Resemblance Nominalism

Another way to account for relations without universals is provided by Resemblance Nominalism. According to this theory, *a* is *F* in virtue of *a* resembling each member of the class of Fs. Armstrong (1989, p. 49–57) has formulated criticisms against this position. However, Resemblance Nominalism was revisited and reformulated by Rodriguez-Pereyra (2002) to address these and other criticisms. In this section, I investigate one of these criticisms and argue that Resemblance Nominalism cannot provide a correct account for relations.

Class nominalists introduce an entity, viz. classes, to explain what properties and relations are. In the last section, the thesis that relations are classes was criticized. In Resemblance Nominalism, properties and relations are not classes. This theory does not need to establish what a property is. No entity is introduced to substitute properties. To account for unary and relational facts, resemblance nominalists use different strategies. Classes are not necessary to explain unary facts. A particular *a* is *F* in virtue of resembling other *F* particulars. However, relational facts are reduced to classes. Consider the relational fact *Rab*. The standard analysis of this fact in Resemblance Nominalism is:

(i) the ordered pair (a, b) resembles other Rxy pairs.

Although Resemblance Nominalism and Class Nominalism adopt different strategies, they rely on the same ontological categories; both theories postulate concrete particulars and classes to solve the problem of universals. However, they exhibit different approaches to the explanation of relations. In Class Nominalism, relations simply are classes. Resemblance nominalists need not commit to this thesis. Since ordered pairs are used by Resemblance Nominalism, I argue that criticisms against Class Nominalism also apply to this solution.

Consider two relational facts: *Pab* and *Sba*. *P* corresponds to *precede*, and *S* corresponds to *succeed*. These are non-symmetric relations. Ordered pairs are required to distinguish relational facts involving *P* from relational facts involving *S*. *Pab* and *Sba* are accounted for as follows:

(ii) the ordered pair (a, b) resembles other Pxy pairs.

(iii) the ordered pair (b, a) resembles other Sxy pairs.

In section (1), two problems with using ordered pairs to explain relations were presented. Both can be applied to Resemblance Nominalism. First, choosing (ii) or (iii) as the correct explanation of *Pab* is arbitrary. Both classes are equally adequate to explain this fact. Second, ordered pairs exhibit an intensional aspect. The ordered pair (a, b) cannot be reduced to particulars *a* and *b* alone.

Ordered pairs could be defined following Kuratowski's proposal. In this way, perhaps Resemblance nominalists could avoid problems involving ordered classes. The relational fact *Pab* could be analyzed in the following way:

(iv) the class {{a}, {a, b}} resembles other classes {{x}, {x, y}} since they satisfy Pxy.

Unfortunately, (iv) does not eliminate the problems arising from the Wiener-Kuratowski procedure. Many definitions of ordered pairs can be formulated. Using a different definition, another analysis of the relational fact *Pab* would emerge. As in Class Nominalism, defining ordered classes through non-ordered classes yields an issue of indeterminacy. For each definition, there will be an alternative ontological account. There is no independent criterion for determining which explanation is correct. Therefore, the way Resemblance Nominalism reduces relations is arbitrary.

The criticism raised by Hochberg also applies to this theory. The Wiener-Kuratowski procedure does not eliminate the notion of order. Compare account (iv) with account (v):

(v) the class {{b}, {a, b}} resembles other classes {{x}, {x, y}} since they satisfy Pxy.

There is no non-arbitrary criterion for choosing (iv) or (v). Both statements could serve as the correct explanation of the relational fact *Fab*. Assuming (iv) or (v) implies the acceptance of a rule to define the order in *Fab*.

Rodriguez-Pereyra (2002, p. 56–62) argues that the problems involving ordered pairs do not pose a challenge for Resemblance Nominalism. He discusses two issues generated by identifying relations with classes: the problem of order and the problem involving the Wiener-Kuratowski procedure.

Rodriguez-Pereyra maintains that the problem of order is not a strong argument against identifying relations with classes. He claims that there is no genuine relation in an ordered pair. Consider the ordered pair definition:

(PO) (x, y) = (u, v) if and only if x=u and y=v.

He asserts that this definition exhausts any notion of order within an ordered pair.

Even if there were an implicit relation in an ordered class, Rodriguez-Pereyra claims that Class Nominalism and Resemblance Nominalism are immune to this criticism, for this implicit relation would not be natural or sparse and solutions to the problem of universals should be concerned only with properties of this kind.

The goal of definition (PO) is to provide identity conditions for ordered classes. Non-ordered classes obey the principle of extensionality. However, this criterion of identity does not apply to ordered classes. The objective of (PO) is not to exhaust the notion of order in any ordered class. On the contrary, this definition makes explicit that an ordered pair cannot be reduced to its elements. The ordered pairs (a, b) and (b, a) are different, even though they have the very same elements.

If there is an implicit relation of order in ordered classes, asserting that this relation is natural is problematic, as there is no precise criterion for its definition. Indeed, this issue cannot be addressed because which properties are natural or sparse is an *a posteriori* matter. However, there is another issue: choosing between (a, b) or (b, a) as part of the explanation of the relational fact *Pab* is arbitrary.

Rodriguez-Pereyra (2002, p. 60) argues that there is no difficulty with choosing (a, b) or (b, a). *Pab* could be reduced in the following ways:

(vi) the ordered pair (a, b) resembles other ordered pairs (x, y) since they satisfy Pxy.

(vii) the ordered pair (b, a) resembles other ordered pairs (x, y) since they satisfy Pxy.

He claims that (vi) and (vii) are equally valid explanations of two relational facts: *Pab* and its converse *Sba*[6]. If we identify *Pab* with (vi), we have adequate reasons to identify *Sba* with (vii). Conversely, it is equally valid to identify P with (vii) and S with (vi). He maintains that, as both analyses are valid, Resemblance Nominalism does not need to decide which is the correct one for a relation and its converse:

> Whether 〈a, b〉's resemblances to other pairs is what makes it true that a bears R to b and 〈b, a〉's resemblances to other pairs is what makes it true that b bears R's converse to a, or the other way round, is a dispute that need not concern Resemblance Nominalism. For either way, what makes a bear R to b and b bear R's converse to a is that certain ordered pairs, among which there are some whose members are a and b, resemble each other (Rodriguez-Pereyra, 2002, p. 60).

If ordered classes are reduced to non-ordered ones, there will be manifold explanations derived from (vi) and (vii) because many definitions of ordered pairs are equally valid. Since (vi) and (vii) are ontologically different, choosing between them is arbitrary. Therefore, Rodriguez-Pereyra's response does not avoid the issue of arbitrariness. Even if both (vi) and (vii) are adequate explanations for *Pab* and *Sba*, the selection is not established by any intrinsic feature of relations *P* and *S*; it is conventional.

Additionally, the problem of the hierarchical structure of reality also arises in the context of the Resemblance Nominalism account. Consider two facts: *Fa* and *Rab*. These

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[6] In a symmetric relation, R and its converse are identical. If P were a symmetric relation, (vi) and (vii) would be equivalent. In such a case, problems related to order would not arise.

facts are composed of a first-order unary property and a binary first-order relation. However, even though both facts are in a first-order level, they are not reduced in the same way within Resemblance Nominalism.

A unary fact has a simple resemblance structure: particulars that resemble each other compose a natural class. But Resemblance Nominalism also has to provide a resemblance structure for relational facts. However, this is not simple. The rule used to account for unary facts cannot be applied in an analysis of relational facts. Compare:

(i) *a* is *F* in virtue of *a* resembling each member of the class of *Fs*;

(ii) *Rab* in virtue of the particulars *a* and b resembling other particulars *x* and *y* since they satisfy Pxy.

(iii) *Rab* in virtue of the class {a, b} resembling other classes {x, y} since they satisfy Pxy.

(iv) *Rab* in virtue of the ordered pair (a, b) resembling other ordered pairs (x, y) since they satisfy Pxy.

(v) *Rab* in virtue of the class {{a}, {a, b}} resembling other classes {{x}, {x, y}} since they satisfy Pxy.

In (ii), the same rule applied to unary facts is extended to binary facts. Clearly, this does not work. Different relations could be explained in the same way, even if it is assumed that all *possibilia* are included in the range of these properties and relations. Resemblance nominalists need to introduce classes to avoid the co-extensionality problem. As in Class Nominalism, this is an arbitrary maneuver.

Which class should we choose to compose relational facts? In (iii), the co-extensionality problem is not avoided even when there are no non-symmetric relations. To avoid these issues, the best option available to Resemblance Nominalism appears to be (iv) or (v).

It is an arbitrary choice to determine which option—(ii), (iii), (iv) or (v)—is the correct explanation for relational facts in Resemblance Nominalism. Why are relational facts composed of ordered classes, whereas unary facts are composed of particulars instead of classes? The way in which the elements of a relational fact are selected to provide the explanation is not uniquely determined. Other rules could be formulated to select these elements.

3. Conclusion

In this paper, I have examined whether reducing relations or relational facts to classes is an appropriate approach. The problem of order and the challenges involving the Wiener-Kuratowski procedure have been discussed in detail. Additionally, the question of how nominalists could exhibit a hierarchical structure of reality has been analyzed. Finally, I have criticized the appeal to classes to explain relations or relational facts, arguing that the selection of the elements of these classes is an arbitrary maneuver.

Recent literature explores other relevant issues involving relations. Nominalist theories seek to reduce relations to particulars to avoid universals. However, some philosophers argue that relations supervene on unary properties. It would not be necessary to postulate relations in addition to monadic properties. Particulars and unary properties would be sufficient to provide an ontological description of the world. Perhaps Nominalism can adopt this reductionist approach to eliminate relations. Naturally, there are criticisms against this thesis. I do not take a stance on which approach is better for Nominalism in analyzing relations. My goal here has been simply to defend that Nominalism cannot reduce relations or relational facts to classes.

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